### Propagation of bistable fronts through a perforated wall

## Hiroshi Matano( 侯野 博 ) Meiji University

In recent years, the behavior of solution fronts of reaction-diffusion equations in the presence of obstacles has attracted attention among many researchers.

In this talk, I will consider the case where the obstacle is a wall with many holes and discuss whether the front can pass through the wall and continue to propagate ("propagation") or is blocked by the wall ("blocking"). The answer depends largely on the size and the geometric configuration of the holes.

This problem has led to a variety of interesting mathematical questions that are far richer than we had originally anticipated. Many questions still remain open. This is joint work with Henri Berestycki and François Hamel.

#### Stability of equilibrium of MHD equations

#### Hideo Kozono(小园英雄) Waseda University

In the 3-dimensional bounded domain D with smooth boundary S, we consider stability problem of solutions to the MHD equations for U = (u, B) denoting the velocity and magnetic field, respectively. Introducing the harmonic vector field

 $H(D) = \{h \mid \nabla h = 0, \nabla \times h = 0 \in D, (h, n) = 0 \in S\}$ 

with n denoting the unit outer normal to S, we first show that  $U_* = (0, B_*)$  with  $B_*$  in H(D) gives an equilibrium state.

Next, we prove that if gard  $B_*$  is small and if the initial disturbance  $U_0$  is sufficiently small in  $L^3(D)$  with  $B_0 - B_*$  perpendicular to H(D), then  $U_*$  is exponentially stable.

This is the joint work with Senjo Shimizu (Kyoto Univ.) and Taku Yanagisawa (Nara Women Univ.).

#### Initial traces and solvability of porous medium equation with power nonlinearity

#### Kazuhiro Ishige(石毛和弘) University of Tokyo

We study qualitative properties of initial traces of solutions to the porous medium equation with power nonlinearity, and obtain necessary conditions on the existence of solutions to the corresponding Cauchy problem. Furthermore, we establish sharp sufficient conditions on the existence of solutions to the Cauchy problem using uniformly local Morrey spaces and their variations, and identify the optimal singularities of the initial data for the solvability of the Cauchy problem. This is a joint work with Nobuhito Miyake and Ryuichi Sato.

# Diffusion-Propagation phenomena of KPP equation: weakly disordered and strongly disordered

## Xing Liang( 梁兴 )

### University of Science and Technology of China

In this talk I will report some work on the Diffusion-Propagation phenomena of KPP equation in almost periodic media. We will mainly show that in weakly disordered media, the similar phenomena with that in ordered media appear. On the other hand, when the media is strongly disordered, some localized propagation phenomena will appear.

## **Invariant Partial Differential Operators in Physics**

Tohru Ozawa(小泽 徹) Waseda University

A characterization is given to the time-dependent free Schrödinger operator by means of the invariance under the Galilei group with gauge, thereby supporting the foundation of Quantum Mechanics on the basis of the invariance naturally arising in Classical Mechanics. This talk is based on a recent joint-work with Hiromichi Nakazato, Department of Physics, Waseda University.

## **Stochastic Transport Maps in Diffusion Models and Sampling**

## Xicheng Zhang(张希承) Beijing Insitute of Technology

In this work, we present a theoretical and computational framework for constructing stochastic transport maps between arbitrary probability distributions using diffusion processes. We begin by proving that the time-marginal distribution of the sum of two independent diffusion processes satisfies a Fokker-Planck equation. Building on this result, we establish the existence and uniqueness of solutions to the associated stochastic differential equation (SDE). Leveraging these theoretical foundations, we develop a method to construct (stochastic) transport maps between arbitrary probability distributions using dynamical ordinary differential equations (ODEs) and SDEs. Furthermore, we introduce a unified framework that generalizes and extends a broad class of diffusion-based generative models and sampling techniques. Finally, we analyze the convergence properties of particle approximations for the SDEs underlying our framework, providing theoretical guarantees for their practical implementation. This work bridges theoretical insights with practical applications, offering new tools for generative modeling and sampling in high-dimensional spaces.

## Exact Morse index of radial solutions for semilinear elliptic equations with critical exponent on annuli

Yasuhito Miyamoto( 宮本 安人 ) University of Tokyo

Let the spatial dimension be greater than or equal to 3.

We study every radial solution of the Henon equation with critical exponent on annuli.

Then it is known that for each n, there exist exatly two radial solutions with n nodal domains.

We are interested in the Morse index of these solutions.

We obtain the exact Morse index provided that the inner radius is close to zero.

Under some special condition on exponents we also obtain the exact Morse index of a positive and negative solution for all annuli.

## **Principal eigenvalue of second order elliptic operator with large advection**

### Maolin Zhou( 周茂林 ) Nankai University

In this talk, we will discuss recent progress on the asymptotic behaviour of the principal eigenvalue of some second order operators. Through analyzing the closed orbits of the advection vector field, we obtain a complete result on how the principal eigenvalue converges for large advection. We also find an unified viewpoint to understand some previous results on this topic, like [Berestycki-Hamel-Nadirashvilli, CMP 2005] and [Chen-Lou, IUMJ2008]. This is a joint work with Shuang Liu and Yuan Lou.

## Quantitative results on time-dependent random conductance models with stablelike jumps

#### Jian Wang(王健) Fujian Normal University

We establish quantitative homogenization results for time-dependent random conductance models with stable-like long range jumps on  $\mathbb{Z}^d$ , where the transition probability from x to y is given by  $w_{t,x,y}|x-y|^{-d-\alpha}$  with  $\alpha \in (0,2)$ . In particular, time-dependent random coefficients  $\{w_{t,x,y}: t \in \mathbb{R}_+, (x,y) \in E\}$  are uniformly bounded from above and below by positive constants and satisfy the finite range dependence property % for time variables in time variable  $t \in \mathbb{R}_+$ , where E is the set of all unordered pairs on  $\mathbb{Z}^d$ . The proofs are based on  $L^2$ -estimates and energy estimates for solutions to regional parabolic equations and multi-scale Poincarl'e inequalities associated with time-dependent symmetric stable-like random walks with random coefficients. This is based on ongoing work with Xin Chen, Zhen-Qing Chen and Takashi Kumagai.

## Stochastic homogenization for non-symmetric jump processes with divergencefree drift

## Xin Chen(陈昕) Shanghai Jiao Tong University

We will study the stochastic homogenization for non-symmetric processes whose infinitesimal operators having the form  $L = L0 + b(\cdot) \cdot \Delta$ , where

$$L_0 f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x))u(z)dz, x \in \mathbb{R}^d$$

for different Lévy measures u(z)dz and div b(x) = 0 for every  $x \in \mathbb{R}^d$ .

## Makeenko-Migdal equations for 2D Yang--Mills: from lattice to continuum

**Rongchan Zhu**(朱蓉禅) Beijing Institute of Technology

In this talk, I will talk about the convergence of the discrete Makeenko--Migdal equations for Yang--Mills model on  $(\setminus eps \setminus Z)^2$  to their continuum counterparts on the plane, in an appropriate sense. The key step in the proof is identifying the limits of the contributions from deformations as the appropriate area derivatives of the Wilson loop expectations.

#### **Random models on regularity-integrability structures**

### Masato Hoshino (星野壮登) Osaka University

In the study of singular SPDEs, it has been a challenging problem to obtain a simple proof of a general probabilistic convergence result (BPHZ theorem). Differently from Chandra and Hairer's Feynman diagram approach, Linares, Otto, Tempelmayr, and Tsatsoulis recently proposed an inductive proof based on the spectral gap inequality by using their multiindex language. Inspired by their approach, Hairer and Steele also obtained an inductive proof by using the regularity structure language. In this talk, we introduce an extension of the regularity structure including integrability exponents, and provide a simpler proof of BPHZ theorem. This talk is based on a joint work with Ismael Bailleul (Université de Bretagne Occidentale).

### Long time behavior of nonlinear stochastic heat equations

#### Kunwoo Kim

Pohang University of Science and Technology(POSTECH)

In this talk, we study nonlinear stochastic heat equations on an interval with periodic boundary conditions, driven by space-time white noise. In the parabolic Anderson model—where there is no drift term and the diffusion coefficient is linear—Gu and Komorowski identified the exact almost-sure Lyapunov exponent and established the central limit theorem as time tends to infinity. Our work identifies conditions on the nonlinear terms which guarantee that a broad family of nonlinear stochastic heat equations exhibits the same long-time behavior as the parabolic Anderson model. This is ongoing joint work with Davar Khoshnevisan and Carl Mueller.

# Fokker-Planck equations of neuron networks: synchronization and dilating the blowup solution

#### Zhennan Zhou ( 周珍楠 ) Westlake University

In this talk, we are concerned with the Fokker-Planck equations associated with the Nonlinear Noisy Leaky Integrate-and-Fire model for neuron networks. Due to the jump mechanism at the microscopic level, such Fokker-Planck equations are endowed with an unconventional structure: transporting the boundary flux to a specific interior point. In the first part of the talk, we present a conservative and positivity preserving scheme for these Fokker-Planck equations, and we show that in the linear case, the semi-discrete scheme satisfies the discrete relative entropy estimate, which essentially matches the only known long time asymptotic solution property. We also provide extensive numerical tests to verify the scheme properties, and carry out several sets of numerical experiments, including finite-time blowup, convergence to equilibrium and capturing time-period solutions of the variant models. Secondly, we introduce a new notion of genealized solutions for this model with a dynamical time rescaling, so that the extension of solutions after blowups becomes possible.

## $\Phi_3^4$ THEORY FROM MANY-BODY QUANTUM GIBBS STATES

#### Xiangchan Zhu (朱湘禅)

## The Academy of Mathematics and Systems Science (AMSS) of the Chinese Academy of Sciences (CAS)

We derive the  $\Phi_3^4$  measure on the torus as a rigorous limit of the quantum Gibbs state of an interacting Bose gas, where the limiting classical measure describes the critical behavior of the Bose gas just above the Bose-Einstein phase transition. Since the quantum problem is typically formulated using a nonlocal interaction potential, a key challenge is to approximate the local  $\Phi_3^4$  theory by a Hartree measure with a nonlocal interaction. This requires uniform estimates on the Hartree measure, which are achieved using techniques from recent development on stochastic quantization and paracontrolled calculus from [GIP15]. The connection to the quantum problem is then established by applying the variational approach in [LNR21], where using a recent correlation inequality from [DNN25] we refine the analysis and derive a quantitative convergence of the quantum correlation functions to those of the Hartree classical field.

## Conservative Dynamic $P(\phi)_2$ Model on $\mathbb{R}^2$ and its Reversible Measures

#### Qi Zhang (张琦) Beijing Institute of Mathematical Sciences and Applications(BIMSA)

In this talk, we consider the conservative dynamic  $P(\phi)_2$  model on the whole plane  $\mathbb{R}^2$  and its reversible measures. We first establish the lattice approximation for both non-conservative and conservative dynamic  $P(\phi)_2$  models on torus  $M\mathbb{T}^2$ . Then we show the tightness of  $P(\phi)_2$  measures on torus  $M\mathbb{T}^2$  by energy estimates. Based on lattice approximation and tightness result, we construct a family of reversible canonical  $P(\phi)_2$  measures under the conservative law. Moreover, we prove the global-well posedness of the conservative dynamic  $P(\phi)_2$  model on  $\mathbb{R}^2$  when the initial value is Tsinghua-Tokyo workshop on PDEs and Probability, March 22-26, 2025

sample from the stationary measure. This is a joint work with Tadahisa Funaki (BIMSA), Bin Xie (Shinshu), and Hang Zeng (BIMSA).

#### Periodic homogenization for non-local stable-like operators

Takashi Kumagai ( 熊谷 隆 ) Waseda University

Homogenization has been a very active area of research in both PDE and probability theory for many years.

In this talk, we will first review classical results on periodic homogenization for divergence form operators. We will then present our recent results on periodic homogenization for non-local, stable-like operators. Both qualitative and quantitative results will be discussed. Moreover, we will explore quantitative periodic homogenization on bounded domains, where the rate of convergence near the boundary slows down.

This talk is based on joint works with X. Chen, Z.-Q. Chen and J. Wang.

## Coupling between Brownian motion and random walks on the infinite percolation cluster

Chenlin Gu ( 顾陈琳 )

Yau Mathematical Sciences Center, Tsinghua University

For the supercritical Z^ d-Bernoulli percolation  $(d \ge 2)$ , we give a coupling between the random walk on the infinite cluster and its limit Brownian motion, such that the typical distance between the paths during [0, T] is of order  $T^{\frac{1}{3}+o(1)}$ . This partially answers an open question posed by Biskup [Probab. Surv., 8:294-373, 2011]. The construction of the coupling utilizes the optimal transport tool, and the analysis relies on local CLT and percolation density concentration. As an application, our result implies the law of the iterated logarithm proved by DuminilCopin [arXiv:0809.4380], and further identifies the limit constant. This talk is based on a joint work with Zhongen Su and Ruizhe Xu.

#### Periodic homogenisation for singular stochastic PDEs

Weijun Xu (许惟均) Peking University

We will introduce renormalisation procedures in recent developments in singular stochastic PDEs, as well as homogenisation problem for equations with oscillatory coefficients. Both renormalisation and homogenisation are singular limiting procedures, but with very different features. It is then natural to ask how these two limiting procedures interact with each other when present in the same problem. We will also share some of our recent works and understandings in this direction. Part of the talk are based on joint works with Yilin Chen and Ben Fehrman.

## Leading term of maximal edge traversal time in first passage percolation with Weibull distribution

## Ryoki Fukushima (福島 龙辉) University of Tsukuba

In this talk, I will discuss the asymptotic behavior of the maximal edge traversal time in first passage percolation. More precisely, consider the random graph obtained by assigning independent and Weibull distributed positive weights ("traversal time") to the edges of the integer lattice. We are concerned with the shortest path connecting the origin to a remote point. The main result gives a sharp asymptotics of the maximal edge traversal time along the shortest path. Based on a joint work with Shuta Nakajima (Meiji University).